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FAST TRACK COMMUNICATION

Readout of solid-state charge qubits using a single-electron pump

C Hines[1], **K Jacobs**[2,3] **and J B Wang**[1]

[1] School of Physics, The University of Western Australia, Perth 6907, Australia
[2] Centre for Quantum Computer Technology, Centre for Quantum Dynamics, School of Science, Griffith University, Nathan 4111, Australia
[3] Department of Physics, University of Massachusetts at Boston, 100 Morrissey Blvd, Boston, MA 02125, USA

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Abstract
A major difficulty in realizing a solid-state quantum computer is the reliable measurement of the states of the quantum registers. In this paper, we propose an efficient readout scheme making use of the resonant tunnelling of a ballistic electron produced by a single-electron pump. We treat the measurement interaction in detail by modelling the full spatial configuration and show that for pumped electrons with suitably chosen energy the transmission coefficient is very sensitive to the qubit state. We further show that by using a short sequence of pumping events, coupled with a simple feedback control procedure, the qubit can be measured with a high accuracy.

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Reliable measurement of qubits is a key issue in the quantum computation, either in the output register [1] or as a central ingredient in the computation [2]. For solid-state qubits, such as quantum dots [3] or Cooper-pair boxes (CPBs) [4], a number of measurement techniques have been proposed. One approach is to place the qubit adjacent to a single-electron transistor (SET), superconducting SET (SSET) or quantum point contact (QPC) [5–17]. In this method, the state of the qubit affects the tunnelling current through the SET or QPC, and this current provides the readout. Much experimental work on this method has been performed by the group of Clark [18, 19], and the measurement of single qubits with an SET has been realized by the groups of Nakamura [20, 21] and Williams [22]. Such a measurement has also been realized by Hayashi *et al* by directly inducing tunnelling from the double dot [23]. However, none of these has yet been realized as a 'single-shot' measurement. Two further methods for measuring a charge qubit have been proposed and realized as single-shot measurements. The first is a scheme by Vion *et al* [24] in which the state of a CPB is converted into a supercurrent.

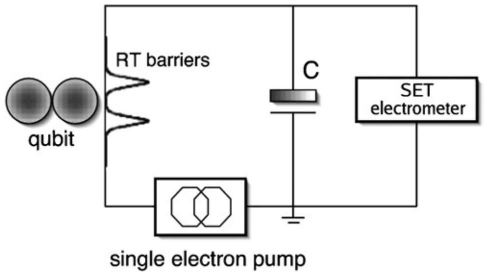

Figure 1. A schematic diagram of the readout system. The electron is pumped through the RT barriers to the capacitor C, the charge on which is measured by an SET electrometer.

The second has been implemented by the group of Martinis *et al* [25, 26] and is a partially destructive measurement in which a tunnelling event is induced by a transition to a third level. A further measurement scheme using a stripline resonator has also been suggested by Sarovar *et al* [27].

Here we consider a readout scheme for a qubit that employs a single-electron pump [28, 29]. In this case, the measurement is obtained by passing a controlled sequence of single-electron pulses through a quantum wire placed adjacent to the qubit. We take the qubit to be formed by a double quantum dot, and in this case we can treat the interaction of the electrons with the system in detail, allowing us to characterize the disturbance to the system. We find that this disturbance does significantly limit the amount of information which can be extracted with the raw measurement. However, because the pump allows us to control the sequence of electron pulses, the qubit may be manipulated by the application of unitary gates between the pulses. We show that, as long as the pumped electrons have a reasonably well-defined momentum, such gates may be applied in a process of feedback so as to correct most of the unwanted disturbance. This provides a near-perfect von Neumann measurement of the qubit within a small number of pulses. We note that sequences of 'pulsed' measurements, and sequential correction using feedback, have also been considered by Jordan *et al* for measurements with a QPC [30, 31].

Our scheme is similar to those involving an (S)SET or quantum point contact, in that the information is extracted as electrons pass by the qubit. However, analyses of these schemes have invariably been performed assuming a simple interaction between the qubit and the probe system. This interaction is assumed to be proportional to the product of a Pauli operator for the qubit and an operator for the probe system [5–15]. The accuracy of this approximation will likely depend on the system parameters (such as the well depth of the quantum dot), and any deviation from this form will in general cause the passing electrons to disturb the system. Since many electrons are required to read out the qubit, a small unwanted disturbance by each electron could potentially impose a significant limit on the measurement. An advantage of our scheme is that the electrons are ballistic, and this allows us to model the Coulomb interaction between the probe electrons and the electron in the quantum dot in detail.

The readout configuration consists of a nanowire connected to a single-electron pump [28, 29] and resonance tunnelling (RT) barriers placed near the qubit, which consists of a single electron in the two coupled double quantum dots separated by a central barrier. The localization of the electron on the right dot (i.e. closer to the wire) represents the state $|1\rangle$, while the localization on the left dot represents the state $|0\rangle$. Figure 1 shows a schematic representation of the configuration. An electron is pumped through the RT barriers to the capacitor C. The charge on C is then measured using an SET electrometer to determine whether the electron was reflected or transmitted by the barriers, and this provides information regarding the charge state of the dot. Since the electron on the capacitor is well separated from the rest of the system, and since its precise state is unimportant, the measurement is robust against any details of the interaction with the SET electrometer, which can perform the charge measurement with a very high accuracy [32]. The single-electron pump produces electrons with a reasonably well-defined and tunable energy, and the device is operated in the ballistic transport regime.

In modelling the system, we include in the Hamiltonian the kinetic energy operators of the electron in the nanowire and the electron in the quantum dots, the Coulomb interaction between the two electrons, the potential of the RT barriers for the electron in the nanowire and the confinement potential for the electron in the quantum dots. Apart from adopting the effective mass and permittivity approximation to account for lattice and screening effects due to the solid environment and considering the spin-related relativistic terms negligibly small, this model includes all perceptible interactions in the system and allows all possible states for the qubit electron in the coupled quantum dots. It is therefore expected to reflect accurately the quantum dynamics of the system during the pumping of the electron across the RT barriers. The measurement of the charge on the electrode at the completion of a pump cycle can be performed with great accuracy [32], and may therefore be described as a von Neumann measurement which distinguishes the sign of the final momentum of the pumped electron.

Details of the model

The joint system composed of the double quantum dot and the electron in the nanowire is described by the wavefunction $\psi(\mathbf{r}_1, \mathbf{r}_2, t)$, where $\mathbf{r}_1 = (x_1, y_1)$ is the position of the electron in the coupled quantum dots, $\mathbf{r}_2 = (x_2, y_2)$ is the position of the electron in the nanowire and t is the time. We will often drop these arguments in the following in order to keep the notation compact. The Schrödinger equation that governs the time evolution of the system wavefunction is

$$\mathrm{i}\hbar \frac{\partial \psi}{\partial t} = \mathcal{H}\psi, \tag{1}$$

where the system Hamiltonian is $\mathcal{H} = \hat{H}_{\text{dot}} + \hat{H}_{\text{wire}} + \hat{H}_{\text{int}}$, $\hat{H}_{\text{dot}} = -\hbar^2/(2m^*)\nabla^2_{\mathbf{r}_1} + V_{\text{dot}}(\mathbf{r}_1)$ is the Hamiltonian for the electron in the coupled dots, $\hat{H}_{\text{wire}} = -\hbar^2/(2m^*)\nabla^2_{\mathbf{r}_2} + V_{\text{wire}}(\mathbf{r}_2)$ is the Hamiltonian for the electron in the wire and $\hat{H}_{\text{int}} = 1/(4\pi\epsilon|\mathbf{r}_1 - \mathbf{r}_2|)$ describes the interaction between the two electrons. The effective mass of the mobile electrons is denoted by m^* and ϵ is the effective permittivity. If, for example, the qubit system and the nanowire detector are built in an AlGaAs/GaAs interface, the effective mass and permittivity are given by $m^* = 0.0667m_e$ and $\epsilon = 12.9\epsilon_0$ [33].

To simulate the evolution of the system we use the Chebyshev–Fourier scheme as detailed in [34]. Briefly, this method approximates the exponential time propagator by a Chebyshev polynomial expansion

$$\psi(t + \Delta t) = \mathrm{e}^{-\mathrm{i}(\mathcal{E}_u+\mathcal{E}_l)\Delta t/2} \sum_{i=0}^{\mathcal{N}} a_i(\alpha) T_i(-i\tilde{\mathcal{H}})\psi(t), \tag{2}$$

where $\mathcal{E}_u$ and $\mathcal{E}_l$ are the upper and lower bounds on the energies sampled by the wavepacket, $\alpha = (\mathcal{E}_u - \mathcal{E}_l)\Delta t/2$, $a_i(\alpha) = 2J_i(\alpha)$ except for $a_0(\alpha) = J_0(\alpha)$, $J_i(\alpha)$ are the Bessel functions of

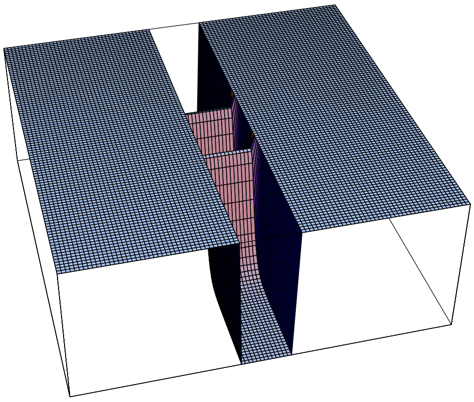

Figure 2. Nanowire with the RT barriers as defined by equation (4).

the first kind, and T_i are the Chebyshev polynomials. To ensure convergence, the Hamiltonian must be normalized according to $\tilde{\mathcal{H}} = [2\mathcal{H} - \mathcal{E}_u - \mathcal{E}_l]/(\mathcal{E}_u - \mathcal{E}_l)$.

We choose the computational basis states of the qubit, $|0\rangle$ and $|1\rangle$, to be mutually orthonormal linear combinations of the lowest two energy eigenstates of the double dot system. Since these states are not the energy eigenstates, the system will oscillate between them, and we must therefore ensure that the duration of the measurement is very small compared to this oscillation time. We model the double dot using the potential

$$V_{\mathrm{dot}}(x_1, y_1) = -V_0 \exp\left(-\frac{m^*}{2V_0}\left(x_1^2 + (y_1 - y_c)^2\right)\omega^2\right) - V_0 \exp\left(-\frac{m^*}{2V_0}\left(x_1^2 + (y_1 + y_c)^2\right)\omega^2\right), \tag{3}$$

where $y_c = 143$ nm, $V_0 = 5.99$ meV and $\hbar\omega = 0.818$ meV describe a typical double dot. The wire RT barriers, as shown in figure 2, are described by

$$V_{\mathrm{wire}}(x_2, y_2) = \frac{v_x}{\cosh^2((x_2 - r)/s)} + \frac{v_x}{\cosh^2((x_2 + r)/s)} + v_y(1 - \Theta(y_2 + d + \delta y_2)\Theta(-y_2 - d + \delta y_2)), \tag{4}$$

where $v_x = 1.09$ meV, $r = 143$ nm and $s = 81.9$ nm, Θ is Heaviside's step function, $v_y \gg v_x$, δy_2 is of a small value representing a very narrow wire and $d = 287$ nm is the distance between the nanowire and the centre of the coupled quantum dots in the y-direction.

Analysis of the measurement

Prior to the measurement, the electron in the double dot and the electron in the nanowire are spatially well-separated, so the state of the combined system is $\rho \otimes |\psi\rangle\langle\psi|$, where ρ is the state of the qubit and $|\psi\rangle$ is the state of the electron incident on the RT barriers. We take the state of this electron to be a Gaussian wavepacket. We include the first four eigenstates of the dot in our numerical simulation, but for the sake of the following discussion, we will assume that the dot contains only the two computational states (which it *very* nearly does). After the interaction, the state of the system is $U\rho \otimes |\psi\rangle\langle\psi| U^\dagger$, where the unitary operator U acts in the joint space, and the qubit and the electron become entangled. For a fictitious observer who performs a von Neumann measurement of the momentum of the (now outgoing) electron, the final state, on obtaining the momentum p, is a normalized version of

$$\begin{aligned}\rho_p &= \langle p| \otimes I(U\rho \otimes |\psi\rangle \langle\psi| U^\dagger) I \otimes |p\rangle \\ &= A_p \rho A_p^\dagger\end{aligned} \tag{5}$$

for some operators A_p which satisfy $\int A_p^\dagger A_p \, \mathrm{d}p = I$, and which completely characterize the measurement process. The probability that the final momentum is p is $\mathrm{Tr}\big[A_p^\dagger A_p \rho\big]$, where this, and all subsequent traces, are taken over the qubit system. Of course, the measurement we actually make determines only whether the final momentum of the qubit is positive or negative. The probability that we will detect a transmission is thus $P_+ = \int_0^\infty \mathrm{Tr}\big[A_p^\dagger A_p \rho\big] \, \mathrm{d}p$, that we will detect a reflection is $P_- = \int_{-\infty}^0 \mathrm{Tr}\big[A_p^\dagger A_p \rho\big] \, \mathrm{d}p = 1 - P_+$ and the corresponding final states are $\rho_+ = \int_0^\infty A_p \rho A_p^\dagger \, \mathrm{d}p / P_+$ and $\rho_- = \int_{-\infty}^0 A_p \rho A_p^\dagger \, \mathrm{d}p / P_-$. We note that the operators A_p are matrices with elements $A_p^{ij} = \langle i| \langle p| U |\psi\rangle |j\rangle$ (where $\{|i\rangle\}$ are a basis for the electron in the double dot) and can therefore be obtained using exclusively pure state simulations.

From the numerical results, we find that the interaction with the qubit causes a very little change in the energy of the pumped electron, namely the scattering is essentially elastic. While we do not assume elastic scattering in obtaining our results, such an assumption is conceptually useful. If the scattering is completely elastic, then one can write the operators as $A_p = \tilde{A}_p |\langle p|\psi\rangle|^2$, where $\tilde{A}_p$ are independent of the initial state. If we select the momentum of the incident electron so that it is sharply peaked at p, then the only operators which contribute to the measurement are those at p and $-p$. In this case the measurement is described solely by the two operators A_p and A_{-p} so that the probabilities are $P_+ = \mathrm{Tr}\big[\tilde{A}_p^\dagger \tilde{A}_p \rho\big]$ and $P_- = \mathrm{Tr}\big[\tilde{A}_{-p}^\dagger \tilde{A}_{-p} \rho\big]$, and the final states are $\rho_+ = \tilde{A}_p \rho \tilde{A}_p^\dagger \, \mathrm{d}p / P_+$ and $\rho_- = \tilde{A}_{-p} \rho \tilde{A}_{-p}^\dagger \, \mathrm{d}p / P_-$.

If the location of the qubit in the double dot has a large effect on the transmission coefficient of the barriers, then a single-measurement cycle (consisting of a pumping event followed by a measurement of the electrode) will extract a lot of information about the state of the qubit in this basis. However, in general, both states give some chance of transmission and a single cycle will not therefore discriminate completely between the basis states. Nevertheless, if the measurement operators A_p are diagonal in this basis, then the measurement will cause no undesirable disturbance. In this case, a complete von Neumann measurement could be approached arbitrarily closely by simply repeating the pump/measurement cycle the required number of times. However, the operators A_p do not have this property, and therefore do cause a disturbance which limits the total amount of information which can be extracted simply by repetition alone.

Using the polar decomposition theorem, we can write the operator A_p as the product of a unitary U_p and a positive operator $P_p = \big(A_p^\dagger A_p\big)^{1/2}$ so that $A_p = U_p P_p$. It is P_p which changes the entropy of the system and thus performs the extraction of information. This information is extracted in the eigenbasis of P_p; that is, it is information about which of the eigenstates of P_p the system is in. The imperfection of the measurement can therefore be understood as resulting because the positive operator P_p is diagonal in the wrong basis, and/or because U_p causes an additional disturbance. Now, if our measurement is described by only two operators, then it is easy to show that Ps for both are diagonal in the same basis. As a result, we can correct for any error in the measurement basis by applying a unitary to select the correct basis. Secondly, upon obtaining the measurement result, since we know which of the Us has been applied (being either U_p or U_{-p}), we can correct for it by applying the Hermitian conjugate after the measurement in what is a simple example of a feedback control

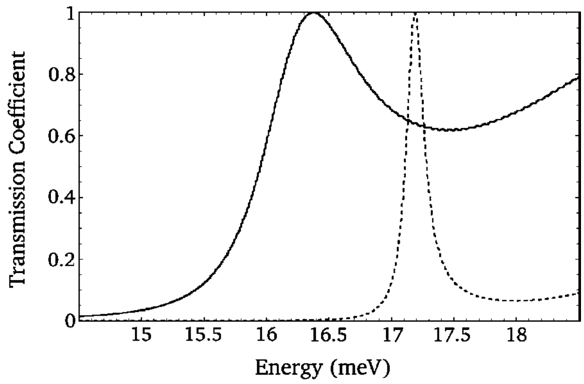

Figure 3. The transmission coefficients of the RT barriers are plotted here as a function of the energy of the pumped electron. The solid and dashed lines give the transmission when the qubit is in the states $|0\rangle$ and $|1\rangle$, respectively.

procedure [35–39]. Therefore, when the measurement has only two operators we can correct completely for any undesirable disturbance and obtain a complete von Neumann measurement by repetition. In our case, such a perfect correction is not possible because the scattering is not completely elastic and the momentum of the electron has a finite spread. Nevertheless, we find that we can improve the performance significantly by using an initial basis change and unitary feedback at each pump/measurement cycle designed to correct for the disturbance of the operators A_p at the centre of the electron wavefunction.

Results

To quantify the accuracy of the measurement we proceed as follows. We encode 1 bit of information in the qubit using the ensemble consisting of the computational basis states chosen with equal probabilities. We then calculate the amount of information which the measurement fails to extract $\mathcal{F} = 1 - M$, where M is the mutual information in bits [40]. This is the residual uncertainty left after the measurement is made. The smaller the $\mathcal{F}$ the more perfect the measurement.

We now examine the transmission profile of the RT barriers as a function of the incident energy for both the states $|0\rangle$ and $|1\rangle$, as shown in figure 3. We find that at energy $E = 16.4$ meV, the RT barriers provide a highly sensitive meter of the qubit state. The residual uncertainty, F, for a measurement using a pumped electron with this average energy is plotted in figure 4. Up to ten repetitions of the pump cycle, both with and without the correcting feedback, are employed. We also calculate this residual uncertainty for a range of values of the energy uncertainty, $\Delta E = 2\%$, 2.8% and 4.2% of the mean energy. Current experiments with electron pumps indicate that spreads at least this narrow should be achievable [28, 29]. The dashed lines give the results without feedback (for six values of the energy uncertainty), and we see in this case that the first pump cycle extracts up to about 90% of the information, and subsequent cycles extract virtually no further information, due to the disturbance caused by the first cycle. However, with the simple feedback procedure described above, which includes an initial rotation, the second pump cycle extracts considerably more information, as do subsequent pump cycles, plateauing at about the sixth cycle. The result is a measurement with high accuracy, in which the residual uncertainty is less than two parts in a thousand. For

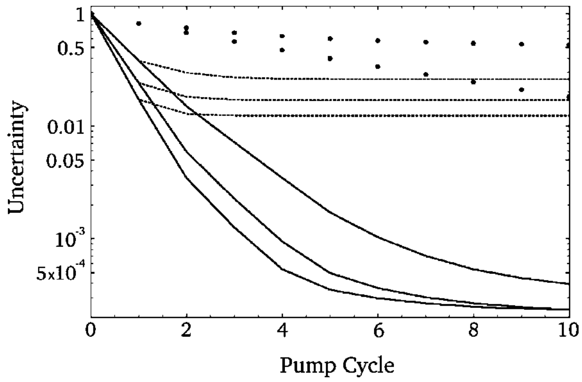

Figure 4. The residual uncertainty is plotted here for a single-qubit measurement using a sequence of pump/measurement cycles. Each curve shows the increase in accuracy as the number of pump cycles is increased. The dashed lines are measurements without feedback, and the solid lines with feedback, both using an incident electron energy of $E = 16.4$ meV. The three curves for each case, from bottom to top, are for incident energy uncertainties of $\Delta E = 2\%$, 2.8% and 4.2%. For comparison, the solid circles represent the residual uncertainty for $E = 17.6$ meV with $\Delta E = 2\%$.

comparison, we also evaluate the measurement obtained when the incident energy is $E = 17.6$ meV with an energy spread $\Delta E = 2\%$. In this case, the transmission is not as sensitive to the qubit state, and ten measurement cycles are not sufficient to reduce the residual uncertainty below 15%.

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References

[1] DiVincenzo D P and Loss D 1998 *Superlattices Microstruct.* **23** 419
[2] Raussendorf R and Briegel H J 2001 *Phys. Rev. Lett.* **86** 5188
[3] Loss D and DiVincenzo D P 1998 *Phys. Rev.* A **57** 120
[4] Makhlin Y, Schön G and Shnirman A 2001 *Rev. Mod. Phys.* **73** 357
[5] Korotkov A N 2001 *Phys. Rev.* B **63** 115403
[6] Aassime A, Johansson G, Wendin G, Schoelkopf R and Delsing P 2001 *Phys. Rev. Lett.* **86** 3376
[7] Buehler T, Reilly D, Brenner R, Hamilton A, Dzurak A and Clark R 2003 *Appl. Phys. Lett.* **82** 577
[8] Jefferson J, Fearn M, Tipton D and Spiller T 2002 *Phys. Rev.* A **66** 042328 1
[9] Kinnunen J, Törmä P and Pekola J 2003 *Phys. Rev.* B **68** 020506 1
[10] Elattari B and Gurvitz S 2000 *Phys. Rev. Lett.* **84** 2047
[11] Wiseman H, Utami D W, Sun H B, Milburn G, Kane B, Dzurak A and Clark R 2001 *Phys. Rev.* B **63** 235308 1
[12] Goan H-S, Milburn G, Wiseman H and Sun H B 2001 *Phys. Rev.* B **63** 125326
[13] Makhlin Y, Schon G and Shnirman A 2000 *Phys. Rev. Lett.* **85** 4578
[14] Clerk A A 2006 *Phys. Rev. Lett.* **96** 056801
[15] Clerk A A, Girvin S M, Nguyen A K and Stone A D 2002 *Phys. Rev. Lett.* **89** 176804
[16] Emiroglua E G, Hasko D G and Williams D A 2003 *Appl. Phys. Lett.* **83** 3942
[17] Averin D V and Sukhorukov E V 2005 *Phys. Rev. Lett.* **95** 126803
[18] Reilly T M B D J, Starrett R P, Greentree A D, Hamilton A R, Dzurak A S and Clark R G 2006 *Appl. Phys. Lett.* **86** 143117

[19] Buehler T M *et al* 2006 *Appl. Phys. Lett.* **88** 192101
[20] Yamamoto T, Pashkin Y A, Astafiev O, Nakamura Y and Tsai J S 2003 *Nature* **425** 941
[21] Pashkin Y A, Yamamoto T, Astafiev O, Nakamura Y, Averin D V and Tsai J S 2003 *Nature* **421** 823
[22] Gorman J, Hasko D G and Williams D A 2005 *Phys. Rev. Lett.* **95** 090502
[23] Hayashi T, Fujisawa T, Cheong H D, Jeong Y H and Hirayama Y 2003 *Phys. Rev. Lett.* **91** 226804
[24] Vion D, Aassime A, Cottet A, Joyez P, Pothier H, Urbina C, Esteve D and Devoret M H 2002 *Science* **296** 886
[25] McDermott R, Simmonds R W, Steffen M, Cooper K B, Cicak K, Osborn K D, Oh S, Pappas D P and Martinis J M 2005 *Science* **307** 1299
[26] Katz N *et al* 2005 *Science* **312** 1498
[27] Sarovar M, Goan H-S, Spiller T P and Milburn G J 2005 *Phys. Rev.* **72** 062327
[28] Likharev K 2001 *Nature* **410** 531
[29] Jehl X, Keller M W, Kautz R L, Aumentado J and Martinis J M 2003 *Phys. Rev.* B **67** 165331
[30] Jordan A N, Korotkov A N and Büttiker M 2006 *Phys. Rev. Lett.* **97** 026805
[31] Jordan A N and Korotkov A N 2006 *Phys. Rev.* B **74** 085307
[32] Martinis J M, Nahum M and Jensen H D 1994 *Phys. Rev. Lett.* **72** 904
[33] Tanner B 1995 *Introduction to the Physics of Elecrons in Solids* (Cambridge: Cambridge University Press)
[34] Wang J and Midglet S 1999 *Phys. Rev.* B **60** 13668
[35] Wiseman H M 1994 *Phys. Rev.* A **49** 2133
[36] Belavkin V P 1999 *Rep. Math. Phys.* **43** 405
[37] Doherty A C and Jacobs K 1999 *Phys. Rev.* A **60** 2700
[38] Doherty A C, Habib S, Jacobs K, Mabuchi H and Tan S M 2000 *Phys. Rev.* A **62** 012105
[39] D'Helon C and James M R 2006 *Phys. Rev.* A **73** 053803
[40] Pierce J R 1980 *An Introduction to Information Theory: Symbols, Signals and Noise* (New York: Dover)